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THE MEASUREMENT OF OCCUPATIONAL MORTALITY

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In making comparisons of the mortality of different occupational groups, it is essential to allow for differences in the age distributions of the groups. The principle of "standardizing" for age is long established. Each of the two conventional methods, direct and indirect, leads to a "Standardized Mortality Ratio" (S.M.R.) which is a weighted average of the ratios of the death rates, by ages, in the occupation, to the corresponding death rates in some standardizing population. In both methods, very great weight is given to the ratio obtained from the oldest age groups. Previous research has shown that there is serious distortion in the recording of occupations, which is greatest among the oldest age groups; the S.M.R.s based on the conventional methods are thus liable to serious error, as well as to "bias".

A more recent method, inverse, of particular use when the age distribution of the occupation is unknown, gives more equal weights to the ratios of the death rates for the various age groups. The S.M.R. calculated by it, therefore, generally differs markedly from those obtained by the conventional methods.

In a fourth, apparently new method, the weighting of the ratios of death rates is again more uniform than in the conventional methods; it has the further advantage of being based on the age distribution of the standardizing population rather than that of the occupation itself as is the case with the third, inverse, method. This "new" method of obtaining an S.M.R., for comparing the mortality of the occupational group as a whole, is therefore the one which is least open to objection on logical grounds.

However, each of the four methods, and of two others given in the literature, is an attempt to compare one occupation with some standard in terms of a single index, averaging the ratios of the death rates by ages. Such an index can take no account of the variation between these ratios and since this variation is often very great none of the methods can fail to be misleading in many cases. Further, the standard deviation of each index takes no account of this variation and so can be misleading also.

The realization that hazards to health vary between occupations dates back at least to Hippocrates, but the measurement of occupational mortality seems to have started with the Census of 1851 in Great Britain (Registrar-General, 1855). The convention of measurement is straightforward. Denominators of mortality rates are obtained from Census Schedules, on which, from 1851 onwards, every member of the population has to be recorded together with his age and occupation. Numerators come from Registrations of Death, which also contain information about age and occupation. The definitions of occupation are nominally exactly the same in these two sources (General Register Office, 1951). Usually, comparisons of occupational mortality are restricted to the ages 20 to 64, and, for men, relate to all males, including both occupied and retired men. (See Benjamin, 1959, for a fuller discussion.)

This paper discusses the measurement of occupational mortality, in terms of the two conventional and some newer methods. The argument is illustrated by the material in Table I, which is taken from the Registrar-General's Decennial Supplement (1958b).

Because of the wide variation in death rates with ages, the differences in the age distribution of the populations have to be allowed for. The principle of "standardizing" for age has long been established and there are two conventional methods, direct and

TABLE 1

POPULATIONS AND DEATHS OF ALL MALES AND COALMINERS IN 1951

Age Group	All Males			Hewers and Getters in Coal-mines			Other Coalminers		
	Census Population (thousands)	Deaths*	Death rate per thousand	Census Population (thousands)	Deaths*	Death rate per thousand	Census Population (thousands)	Deaths*	Death rate per thousand
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
20 to 24	1,427·2	1,973·4	1·38	13·6	24·4	1·79	26·3	43·8	1·67
25 to 34	3,139·7	5,003·6	1·59	52·3	97·8	1·87	71·8	129·0	1·80
35 to 44	3,290·4	9,437·2	2·87	50·6	184·0	3·64	83·8	268·8	3·21
45 to 54	2,874·3	23,604·0	8·21	39·0	430·6	11·05	87·4	689·6	7·89
55 to 64	2,028·1	46,550·4	22·95	21·1	870·0	41·20	62·9	1,307·8	20·80
20 to 64	12,759·8	86,568·6	6·78	176·6	1,606·8	9·10	332·1	2,439·0	7·34

*One-fifth of the deaths registered over the five years 1949-53.

indirect. The term "Standardized Mortality Ratio" strictly refers only to the measure produced by the indirect method, but for convenience in this paper the various measures discussed are all called S.M.R.s and are differentiated by indicating the method used.

Methods of Obtaining Standardized Mortality Ratios

In the direct method, the death rates for one particular occupation are applied to the population of all males, to obtain the number of deaths which would have occurred in the all males population, and this number is compared with the number of deaths actually observed for all males. Thus, from Table 1, for Hewers and Getters, the figures of column (2) are multiplied by those of column (7) to give

1,427·2 × 1·79 + .. + .. + .. + .. = 135,722

deaths which would have occurred compared with 86,568·6 recorded. The ratio of these figures, expressed as a percentage (*i.e.*, 100 × 135,722 ÷ 86,569 =) 157, is called here the S.M.R. (direct). The usual name for this ratio is Comparative Mortality Figure (Registrar-General, 1938).

In the indirect method, the death rates for all males are applied to the population of the particular occupation, to obtain expected deaths, which are related to the deaths observed. Thus from Table 1, for Hewers and Getters, the figures of column (5) are multiplied by those of column (4) to give

13·6 × 1·38 + .. + .. + .. + .. = 1,052

deaths to be compared with 1,606·8 recorded. The ratio of these figures, expressed as a percentage (*i.e.*, 100 × 1,606·8 ÷ 1,052 =) 153, is the S.M.R. (indirect).

Recently, Kerridge (1958) has introduced to this country* a measure which he calls the inverse S.M.R. This method is to divide the death rates for all males into the numbers of deaths observed in the particular occupation to get an estimate of the population in the occupation which should have existed to "justify" the number of deaths observed. Thus, from Table 1, for Hewers and Getters, the figures of column (6) are divided by those of column (4) to give

24·4 ÷ 1·38 ÷ .. + .. + .. + .. = 233.7

thousand which would have been the size of the population to justify these deaths, whereas there were only 176·6 thousand actually in the population. The ratio of these two figures expressed as a percentage (*i.e.*, 100 × 233·7 ÷ 176·6 =) 132, is the S.M.R. (inverse).

Kerridge pointed out that the standard error of the inverse S.M.R. is generally greater than that of the conventional ratios (see Appendix for demonstration), and so it might be thought that the differences between the inverse S.M.R. and the others were explained in this way. In Table 2, the S.M.R.s both for Hewers and Getters and for Other Coalminers are given, together with their standard errors calculated on the usual assumption that the variance of the number of deaths in any one age group of the particular occupation is equal to the number of deaths.

TABLE 2

STANDARDIZED MORTALITY RATIOS FOR HEWERS AND GETTERS AND FOR OTHER COALMINERS IN 1951

Method of Standardization	Hewers and Getters		Other Coalminers	
	S.M.R.	Standard Error	S.M.R.	Standard Error
Direct	157	4·0	96	2·0
Indirect	153	3·8	96	1·9
Inverse	132	5·1	105	3·3

Here it can be seen that the standard errors of the inverse S.M.R.s are indeed the highest but that they are not sufficiently high to account for the difference of over 20 points in the S.M.R.s for Hewers and

*Kerridge (1959) has pointed out that, although his method was developed independently, it had been published previously in America (Doering and Forbes, 1939).

Getters. Further the inverse S.M.R. for Other Coalminers indicates mortality experience worse than average whereas the conventional measures indicate experience better than average. It is thus desirable to examine the various methods to see why they differ and to determine a basis for deciding which is the most appropriate of them for any particular purpose.

Let the death rate in one age group in a particular occupation, expressed as a percentage of the death rate in the same age group in the population of all males, be called θ. For Hewers and Getters, the values of the θs are obtained from Table 1 by dividing the figures of column (7) by those of column (4), and multiplying by 100, to give (100 × 1·79 ÷ 1·38 =) 130, etc.; for Other Coalminers from column (10) and column (4) to give (100 × 1·67 ÷ 1·38 =) 121, etc. These values are given in columns (6) and (9) of Table 3 below.

In the Appendix it is shown that each of the three methods is equivalent to obtaining a weighted average of the θs. Table 3 also gives the weights for the S.M.R.s of Hewers and Getters and of Other Coalminers (obtained from the material of Table 1).

The weightings of the θs for the direct method are by the deaths amongst all males, in the age groups, which are given in column (3) of Table 3. It can be seen that over half the weight (53·8%) is given to the θ for the oldest group, and another quarter of the weight (27·3%) to the θ for the next oldest group. These S.M.R.s therefore reflect the mortality experience of the oldest men and take practically no account of that of younger men. Since the θs for the two oldest groups of Hewers and Getters, for example, are considerably higher than those for the younger groups, this leads to bias. Similarly, with Other Coalminers, there is a bias in the opposite direction.

For the indirect method, the weightings of the θs are by the expected deaths in the occupations which are given in columns (5) and (8). It so happens that these distributions are similar to those of column (3) and therefore the same effects are observed and the S.M.R.s calculated by the two conventional methods are closely similar: in other occupations the distributions of expected deaths by age may be rather different, but normally by far the greatest weights will be given to the θs for the oldest groups.

For the inverse method, the weightings are by the age distributions of the populations in the occupations, which are given in columns (4) and (7) respectively. These distributions differ appreciably, but are both much more evenly spread over the age groups than are the deaths. Thus for each occupation the θs get comparatively equal weight and the S.M.R. lies more or less in the middle of the range of θs.

A fourth method, which does not appear to have been discussed before, suggests itself. In this the θs are weighted by the age distribution of all males as given in column (2). It is calculated by multiplying the population of all males in each age group by the ratios of the death rates and dividing by the total population of all males. Thus, from Table 1, for Hewers and Getters, the figures of column (2) are multiplied by those of column (7) and divided by those of column (3) to give

$$1{,}427{\cdot}2 \times 1{\cdot}79 \div 1{\cdot}38 + .. + .. + .. + .. = 17{,}226$$

The ratio of this figure to that for the all males population, expressed as a percentage (*i.e.*, 100 × 17,226 ÷ 12,760 =) 135, is the S.M.R. ("new" method). Since the weightings of the θs are by the age distribution of a population, they are broadly similar to those used in the inverse method and the S.M.R.s can be expected to lie in the middle of the spread of the individual θs. They turn out to be 135 for Hewers and Getters and 106 for Other Coalminers, which compare with S.M.R.s (inverse) of 132 and 105 respectively.

Two other broadly comparable methods exist. That of Yule (1934), using the "equivalent average death rate", is, in effect, a weighting of the θs by the population death rates in the age groups; that

TABLE 3

DEATH RATES AS PERCENTAGES OF RATES FOR ALL MALES (θs) AND THEIR WEIGHTS IN S.M.R.s
(Distributions by age of populations, deaths, etc.)

Age Group	All Males		Hewers and Getters			Other Coalminers		
	Population (%)	Deaths (%)	Population (%)	Expected Deaths (%)	Death Rates as % of Rates For All Males (θ)	Population (%)	Expected Deaths (%)	Death Rates as % of Rates For All Males (θ)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
20 to 24	11·2	2·3	7·7	1·8	130	7·9	1·4	121
25 to 34	24·6	5·8	29·6	7·9	118	21·6	4·5	113
35 to 44	25·8	10·9	28·6	13·8	127	25·2	9·4	112
45 to 54	22·5	27·3	22·1	30·4	135	26·3	28·1	96
55 to 64	15·9	53·8	12·0	46·0	180	18·9	56·6	91
20 to 64	100	100	100	100	134	100	100	108

1 2 3 4 5 6 7 8 9

TABLE 4

RATIO OF DEATH RATES IN OCCUPATIONS TO DEATH RATES FOR ALL MALES

Registrar-General's Large Occupational Groups		Population Aged 20-64 (thousands)	Values of θ					Range of θ	S.M.R. (Indirect)
No	Brief Description		20-24	25-34	35-44	45-54	55-64		
1	Farmers, etc.	230·1	154	94	79	67	69	87	70
2	Gardeners, etc.	145·4	107	111	88	80	76	35	79
3	Other agricultural workers	334·9	104	96	84	70	72	34	76
4	Coal face getters, etc.*	197·9	128	117	124	130	174	57	148
5	Other Coalminers, underground†	213·8	111	108	112	94	87	25	93
23	Turners, etc.	165·0	66	59	80	78	90	31	81
25	Precision fitters, etc.	172·1	73	76	82	87	92	19	87
26	Machine erectors, etc.	440·2	109	108	103	105	117	14	111
29	Plumbers	72·7	90	74	85	92	103	29	94
33	Electrical fitters	99·3	118	126	136	131	138	26	133
34	Electricians	107·1	93	97	96	112	106	19	105
37	Welders, etc.	57·5	112	109	107	113	107	6	110
54	Carpenters, joiners	199·7	101	89	93	87	93	14	91
58	Builders, etc. labourers	135·6	117	153	174	146	139	57	145
59	Bricklayers	121·6	98	86	89	87	103	17	94
62	Other building, etc. workers	269·9	32	46	55	56	57	26	56
63	Other painters, etc.	221·9	104	99	103	104	108	9	106
72	Drivers of road vehicles	509·5	99	97	92	91	87	12	90
81	Salesmen, etc. (food goods)	68·0	64	72	77	87	78	23	79
107	Clerks, typists	780·2	82	94	105	110	98	28	101
108	Warehousemen, etc.	212·9	90	100	107	110	108	20	108
109	Crane, etc. drivers	102·0	157	106	107	93	93	64	96

*Occupations coded 041 and 042, whereas in Table 1, "Hewers and Getters" includes those coded 042 only.
†In Table 1, "Other Coalminers" includes coalminers, above ground, also.

of Yerushalmy (1951) weights the θs equally. These methods are equivalent, therefore, to the direct method and the "new" method, respectively, provided that the standardizing population is taken with an even distribution over the whole age range. They are not considered further below.

Most discussions on the measurement of mortality have been in terms of comparisons between districts, and methods based on the use of Life tables (see, for example, Benjamin, 1959) have been found suitable for some purposes. They are not considered here partly because they do not appear to have any major advantages for occupational comparisons but mainly in view of the conclusions of this study.

Reliability of the Measures

The standard errors given in Table 2 take account only of the variation in the numbers of deaths, *i.e.*, of the accuracy of the individual values of θ. They do not indicate at all the variation between the values of θ, which is seen in Table 3 to be considerable for both mining occupational groups. An index of this variation can be obtained from the range of the values of θ, that is, from the difference between the largest θ and the smallest θ. The S.M.R.s obtained by the four methods and the ranges of the θs are given below:

	Hewers and Getters	Other Coalminers
S.M.R. (direct)	157	96
S.M.R. (indirect)	153	96
S.M.R. (inverse)	·132	105
S.M.R. (new method)	135	106
Range of θs	62	30

The S.M.R.s calculated by the two conventional methods will be essentially different from those calculated by the two newer methods unless either the age distributions of the deaths and of the population are similar or all the values of θ for one occupation are similar. The first of these situations used to arise when the population was much younger and the variation in death rates with ages much less marked: in 1851, these two factors operated to such an extent that the weights for use in the direct method would have been 11·8%, 20·9%, 21·0%, 21·9% and 24·4%. That the second of these situations arises only seldom is illustrated in Table 4. This gives the values of θ, taken from the Decennial Supplement (Registrar-General, 1958b) for all 22 occupational groups which had at least 50 deaths recorded in each age group (servicemen and policemen were not considered).

It can be seen that for most occupational groups the values of θ vary markedly with age: in only two groups is the range of the θs less than 10 and in just over a half of the occupations there are θs both below and above 100. It is of interest that in many occupations there is a pronounced age gradient.

Discussion

The present concept of occupational mortality is subject to fundamental objections which arise over selection and retirement; (see, for example, Reid (1959)). Any arduous job can only be performed by fit men and it seems inherently likely that such men, in their youth, will have more favourable mortality experience than men who are not fit enough to carry out such heavy work. Similarly,

the more arduous the job the earlier men must retire from it: coal-face work, for example, cannot be carried on at the same pace as formerly by men of over, say, 45. Such men may be employed in some lighter occupation, after retiring from their arduous work, until they die, and there is clearly a possible temptation both to these men when they complete Census schedules and to their widows registering their deaths, to record their earlier main occupation. That this led to serious inaccuracy in the assessment of mortality in mining occupations in 1951 has been shown by Heasman, Liddell, and Reid (1958). From their material it was clear that the greatest distortions occurred amongst the oldest age groups.*

It is thus clear that to assess occupational mortality by giving almost the entire weight to these groups will often lead to unreliable comparisons. Thus there is an important argument against the use of the conventional methods of standardization. The two newer methods (inverse and "new") are both better from this point of view. As in the "new" method the weights are determined by the age distribution in the population for all males and hence are standard for each occupation, the S.M.R.s obtained in this way are to be preferred to inverse S.M.R.s. The choice of method may, however, be governed largely by what information is available; for example, Kerridge (1958) suggested the use of the S.M.R. (inverse) where the age distribution in the occupation is unknown. Another factor of importance is the relative ease of computation.

A further possibility of improving the reliability of comparisons is to restrict still further the age range considered. Based on ages 20 to 54 the four methods produce S.M.R.s which are in much closer agreement. Even this, however, does not overcome the problems which arise over selection into the occupations: this might suggest curtailment at the younger end also, which would support Yule's (1934) comment that because there are so few deaths in the younger age groups, their inclusion can lead to serious inaccuracy. In any case, no single index can fail to be misleading in many cases. The American Public Health Association (1951) refers to "fantastic fallacies which may be inherent" when the conventional methods are used to compare mortality of populations with differing age distributions. Clearly, there is no way of describing in one term both the average of the relative death rates (θs) and their variation, which is usually considerable. It is therefore essential, as implied by Reid (1959) and many others, to examine each column of θs separately; although the newly proposed method of calculating an S.M.R. appears to be the most soundly based, neither it, nor any other single mortality index, should be used without a knowledge of the inadequacy inherent in it because of the variation in relative death rates.

*No allowance for these distortions has been made in the material discussed in this paper, which has been used purely for illustrative purposes

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APPENDIX

The notation used is indicated in the table below:

Age Group	All Males			Particular Occupation		
	Population	Death Rates	Deaths	Population	Death Rates	Deaths
1 2 · · i	$P_i = \alpha_i P$	R_i	$X_i = \alpha_i P R_i$	$p_i = \beta_i p$	$r_i = \lambda_i R_i$	$x_i = \beta_i p \lambda_i R_i$
All	P	—	$\Sigma(X_i) = \Sigma(\alpha_i P R_i)$	p	—	$\Sigma(x_i) = \Sigma(\beta_i p \lambda_i R_i)$

$P_i = \alpha P$ $\quad X_i = \alpha P R_i$ $\quad p_i = \beta p$

$x_0 = \beta N_0$

Capitals are used for All Males, lower case letters for the particular occupation. The αs and βs give the proportions of the populations in each age group and so

$$\Sigma(\alpha_i) = 1 \; ; \; \Sigma(\beta_i) = 1 .$$

The ratios of the death rates (r_i) in the particular occupation to the death rates for all males in the same age groups (R_i) are λ_i. It is these values, expressed as percentages, which are quoted in columns (6) and (9) of Table 3, where they are called θs.

The S.M.R.s (expressed as ratios and not as percentages) calculated by the three methods are found to be:

$$\textit{Direct} \quad A = \frac{\Sigma(P_i r_i)}{\Sigma(X_i)} = \frac{\Sigma(\alpha_i P \cdot \lambda_i R_i)}{\Sigma(\alpha_i P \cdot R_i)} = \frac{\Sigma(\alpha_i R_i \cdot \lambda_i)}{\Sigma(\alpha_i R_i)}$$

$$\textit{Indirect} \quad B = \frac{\Sigma(x_i)}{\Sigma(p_i R_i)} = \frac{\Sigma(\beta_i p \cdot \lambda_i R_i)}{\Sigma(\beta_i p \cdot R_i)} = \frac{\Sigma(\beta_i R_i \cdot \lambda_i)}{\Sigma(\beta_i R_i)}$$

$$\textit{Inverse} \quad C = \frac{\Sigma(x_i/R_i)}{p} = \frac{1}{p} \Sigma\left(\frac{\beta_i p \cdot \lambda_i R_i}{R_i}\right) = \Sigma(\beta_i \cdot \lambda_i)$$

and the fourth measure is defined as

"*New*" $\quad D = \Sigma(\alpha_i \cdot \lambda_i)$.

Each of the four measures is seen to be a weighted mean of the λ_i. (When expressed as percentages, they are weighted means of the θs.) The weightings are according to

Direct	A: $\alpha_i R_i$	*i.e.*, according to deaths of All Males.
Indirect	B: $\beta_i R_i$	*i.e.*, according to expected deaths in the particular occupation.
Inverse	C: β_i	*i.e.*, according to the population in the particular occupation.
"*New*"	D: α_i	*i.e.*, according to the population of All Males.

On the usual assumptions (Registrar-General, 1958a; Kerridge, 1958) that (*a*) the number of deaths in each age group of the particular occupation is distributed with variance equal to mean and independently of the number of deaths in the other age groups and (*b*) the other terms (P_i, p_i, X_i and hence α_i, β_i, R_i) are based on such large numbers as to have effectively zero variance, we have

$$\text{var}(x_i) = x_i$$

i.e. $\quad \text{var}(\beta_i p \cdot \lambda_i R_i) = \beta_i p \cdot \lambda_i R_i$.

Thus $\quad \text{var}(\lambda_i) = (\beta_i p \cdot \lambda_i R_i)/(\beta_i p R_i)^2$

$$= \lambda_i/(\beta_i R_i p).$$

For any weighted mean $(\Sigma w_i \lambda_i)/(\Sigma w_i)$ of the λ_i in which the weights w_i obey the conditions assumed above

$$\text{var}\left\{\frac{\Sigma(w_i \lambda_i)}{\Sigma w_i}\right\} = \frac{\Sigma[w_i^2 \text{ var}(\lambda_i)]}{[\Sigma(w_i)]^2}$$

Hence the variances of the four measures are

$$\textit{Direct} \quad \text{var}(A) = \frac{1}{p} \frac{\Sigma(\alpha_i^2 R_i \lambda_i \beta_i)}{[\Sigma(\alpha_i R_i)]^2}$$

$$\textit{Indirect} \quad \text{var}(B) = \frac{1}{p} \frac{\Sigma(\beta_i R_i \lambda_i)}{[\Sigma(\beta_i R_i)]^2}$$

$$\textit{Inverse} \quad \text{var}(C) = \frac{1}{p} \Sigma(\beta_i \lambda_i / R_i).$$

"*New*" $\quad \text{var}(D) = \frac{1}{p} \Sigma(\alpha_i^2 \lambda_i / \beta_i R_i).$

A simple comparison of the variances is unrewarding, but in a number of cases it is possible to compare in terms of the squares of the coefficients of variation. In comparing the two conventional measures, direct (A) and indirect (B), we find

$$\left\{\frac{\text{C.V. (A)}}{\text{C.V. (B)}}\right\}^2 = \frac{\text{var}(A)}{A^2} \frac{B^2}{\text{var}(B)}$$

$$= \frac{\Sigma(\alpha_i^2 R_i \lambda_i/\beta_i) \; \Sigma(\beta_i R_i \lambda_i)}{[\Sigma(\alpha_i R_i \lambda_i)]^2}$$

on reduction, and this can be shown to be

$$= \frac{\text{A.M. } [(\alpha_i/\beta_i) \text{ weighted by } \alpha_i R_i \lambda_i]}{\text{H.M. } [(\alpha_i, \beta_i) \text{ weighted by } \alpha_i R_i \lambda_i]}$$

$$\geqslant 1$$

Hence, the S.M.R (direct) has a higher coefficient of variation than the S.M.R. (indirect), except when all $\alpha_i = \beta_i$. If the age distribution of the particular occupation is similar to that of All Males, the ratio of the coefficients of variation will be close to unity.

It is also possible to compare the S.M.R. (inverse), C, with the S.M.R. (indirect), B. Here the ratio of the squares of the coefficients of variation reduces to

$$\left\{\frac{\text{C.V. (C)}}{\text{C.V. (B)}}\right\}^2 = \frac{\Sigma(\beta_i \lambda_i \cdot 1/R_i)}{\Sigma(\beta_i \lambda_i)} \frac{\Sigma(\beta_i \lambda_i R_i)}{\Sigma(\beta_i \lambda_i)}$$

$$= \frac{\text{A.M. } (R_i \text{ weighted by } \beta_i \lambda_i)}{\text{H.M. } (R_i \text{ weighted by } \beta_i \lambda_i)}$$

$$\geqslant 1$$

Hence, the S.M.R. (inverse) has a higher coefficient of variation than the S.M.R. (indirect) unless all the death rates by ages for All Males, the R_i, are equal. Since these rates vary markedly (*cf.* column (4) of Table 1), the differences between their Arithmetic and Harmonic means will be considerable and so the two S.M.R.s can be expected to have noticeably different coefficients of variation. The ratios for Hewers and Getters and for Other Coalminers can be obtained from Table 2 as follows:

	Hewers and Getters	*Other Coalminers*
Direct compared with Indirect $\dfrac{\text{C.V. (A)}}{\text{C.V. (B)}}$	1·03	1·05
Inverse compared with Indirect $\dfrac{\text{C.V. (C)}}{\text{C.V. (B)}}$	1·56	1·59

The two lower ratios are very much smaller than that obtained from the data used by Kerridge. Here

$$\frac{\text{C.V. (C)}}{\text{C.V. (B)}} = \frac{6{\cdot}4}{2{\cdot}4} \Big/ \frac{1{\cdot}4}{6{\cdot}7} = 4{\cdot}1$$

and this high value is due to the wide variation between the R_i which arises because the age range is greater than is normally considered in occupational mortality comparisons.